PARAMETERS OF A STATIONARY RADIALLY
SYMMETRICAL JET OF VAPORS HEATED
BY LASER RADIATION

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UDC 532.529.5/6

The solution [1] of the problem of the stationary radially symmetrical movement of vapors heated by monochromatic radiation is generalized for the case of an arbitrary (tabular) dependence of the coefficient of absorption and the adiabatic index on the temperature and density. The calculations of thermodynamical and optical properties of vapors of a number of elements carried out in a wide range of densities and temperatures and the solution referred to made it possible to determine the parameters of a stationary jet of vapors in a wide range of radiation flux densities and characteristic dimension. Some results of the calculations for carbon and aluminum are presented. It turns out that a characteristic property of the distribution pattern of parameters in a jet of vapors is the presence on the surface of a zone of cold vapors and a zone of their heating — the heating wave front. However, for large radiation flux densities the extent of the zone of cold vapors is not large. A rough estimate of the intensity of reradiation of the heated vapors is derived. It is shown that for characteristic dimensions of the vapor layer on the order of 0.3-1 cm the intensity of reradiation can be high enough that the pattern of movement found without considering reradiation can change somewhat. It is shown that the solution examined can be generalized also to the case where the transfer of energy by radiation of the continuous spectrum is taken into account.

A jet of vapors is formed from the effect of laser radiation on the surface of a solid body. For a sufficiently large flux density of the incident radiation the pressure at the surface and, with it, the temperature of the phase transition increase enough that the vapors become weakly ionized and absorb the laser radiation with free—free electron transitions in the field of ions and neutral atoms. As a result the vapors are strongly heated and ionized. The absorption by ionized vapors can be so great that only a small part of the radiation energy goes to the vaporizing surface (the "screening" of this surface takes place), while the rest of the energy is absorbed by the vapors.

The possibility of vaporizing practically any substance and (as a result of absorption of radiation in the vapors) heating it to very high temperatures and (with the dispersion of the vapors into a vacuum) accelerating it to high velocities allows one to carry out a whole series of physical and gas dynamic experiments. It is therefore of undoubted interest to determine the parameters of the jet of vapors.

The picture of the heating process and the movement of vapors can be very complicated. It can be fully described only through a solution of the nonstationary gas dynamic problem taking into account the release of energy during the absorption of radiation and a whole series of physical processes. For a long enough duration of energy input or for a small size of the "spot" on which the incident radiation is focussed, effects of lateral expansion of the vapors appear and the problem becomes two-dimensional and in some cases even three-dimensional.

At the same time it is desirable to simplify the situation in order to conduct experimental studies in the plasma which is formed and on the process of the radiation effect itself. In this sense it is attractive

to use the stationary system of heating and vapor movement indicated in [1]. In fact, suppose the radiation falls on the surface of a sphere from all sides or in some cone (with the apex at the center of the sphere) limited in divergence and bearing on the contour of the irradiation "field" on the surface of the sphere. Suppose the rays fill the entire cone and are directed radially toward the center of the sphere. Although the latter, strictly speaking, is impossible, nevertheless if the radius is somewhat larger than the minimum size which can be achieved in focussing the bundle of rays leaving the lens, then in practice the radiation can be considered as falling perpendicularly on the surface of the sphere. It is natural that for this geometry the movement of vapors from the vaporizing sphere will also be radially symmetrical.

Over a long enough time, after the heating of vapors occurs (the rather complicated pattern of this process is described in [2, 3]), when the thickness of the layer of vapors becomes comparable to the dimension r_0 of the irradiated sphere, the density ρ of the vapors begins to drop faster than in the plane case. In the region of multiple ionization it can be assumed approximately that the coefficient of absorption of optical radiation $\varkappa$ does not depend on the temperature T but varies only with the density (approximately as ρ^β, $\beta \approx 0.5\text{--}1$).

Therefore the more rapid drop in density leads to an increase in the transparency of the peripheral layers of the vapors. The absorption occurs principally at a distance of $(r-r_0) \lesssim r_0$, and the optical thickness of this zone proves to be constant. As a result it is possible for a stationary state of movement of the vapors heated by radiation to develop in a "straight-walled nozzle." A transition through the speed of sound takes place in some critical section of radius r_* (the parameters in this section will be marked with asterisks).

In [1] the problem was solved (the parameters are determined in the critical section and their distribution with respect to the vaporizing surface is found) for the case where the coefficient of absorption was described by an exponential function of the internal energy e and the density ρ:

$$\varkappa = Ke^{-\alpha}\rho^\beta \qquad (0.1)$$

while the adiabatic index γ was taken as constant.

In such a case the movement has a unique isomorphic nature: The pattern of the movement can be described by a single (for each α, β, and γ) dimensionless distribution of p/p_*, ρ/ρ_*, u/u_*, and q/q_* with respect to r/r_* (u is the velocity of movement, q is the radiation flux density), while the parameters in the critical section vary in an exponential fashion with a change in r_* and q_* and consequently in r_0 and q_0 (q_0 is the radiation flux density at the surface of the sphere in the absence of screening).

At the same time it is clear that (0.1) cannot describe the actual dependence $\varkappa(e, \rho)$ in the whole range of e and ρ. In fact, it was already shown in [4] that in problems on the distribution of a vaporization wave under the effect of radiation (taking into account the screening by vapors) there is always a cold layer at the vaporizing surface which has a temperature close to the phase transition temperature T_v. But the gas will be weakly ionized in this layer for rather low vapor pressures and corresponding phase transition temperatures T_v. Therefore the coefficients of absorption vary here, roughly speaking, according to the Boltzmann exponential law, i.e., sharply, and they depend in an essentially nonlinear fashion on the temperature T and are almost independent of the density ρ. Thus, it can be said that the cold layer at the surface, where (0.1) is invalid, can have a great enough extent so that either the pattern of distribution of the parameters changes as compared to that presented in [1] or it generally leads to the impossibility of a stationary state. Let us note that just as in the plane case (when the thickness of the vapor layer is still small in comparison with the characteristic dimension r_0) the pattern of movement can differ markedly from that which is given by the isomorphic solution [5, 6] found on the assumption of (0.1). In such a case the pattern of movement can be obtained only through numerical calculations of the nonstationary problem, similar to [2, 3] drawing on tabular dependences of $\varkappa(e, \rho)$ and $\gamma(e, \rho)$. Moreover the stationary solution [1] is easy to generalize to the case of a variable adiabatic index γ and an arbitrary dependence of $\varkappa$ on e and ρ. This allows a rather direct determination of the parameters of the vapor jet as a function of the incident flux density q_0 and the characteristic dimension r_0 taking into consideration the actual equation of state and the actual dependence of the absorption coefficient on the temperature and density.

1. The system of equations describing the process of movement and heating has the form

$$dp + \rho u\, du = 0 \qquad (1.1)$$

$$\rho u r^{\nu-1} = \rho_* u_* r_*^{\nu-1} = \dot{M} / \zeta(\nu) \qquad (1.2)$$

$$\dot{M}\left(h + \tfrac{1}{2}u^2\right) + F = \dot{M}\left(h_* + \tfrac{1}{2}u_*{}^2\right) + F_* \tag{1.3}$$

$$dF^-/dr = + F^-\varkappa\rho \qquad (F^- < 0) \tag{1.4}$$

$$dF^+/dr = - F^+\varkappa\rho \qquad (F^+ > 0) \tag{1.5}$$

Here $h = e + p/\rho$ is the enthalpy, M is the mass flow rate, F is the total radiant energy flux ($F = F^+ + F^-$), F^- is the energy flux reaching the surface, F^+ is the radiant flux reflected from the vaporization wave, and the equations at the vaporization wave (index w) have the form

$$\rho_w u_w r_w^{\nu-1} = \dot{M}/\zeta(\nu) \tag{1.6}$$

$$p_0 = p_w + \rho_w u_w{}^2 \tag{1.7}$$

$$\dot{M}\left(h_w + \tfrac{1}{2}u_w{}^2\right) + F_w = -\dot{M}Q_v \tag{1.8}$$

$$F_w{}^+ = (K_r)_w F^-_w \tag{1.9}$$

$$\zeta(\nu) = 1, 2\pi, 4\pi \quad \text{for} \quad \nu = 1, 2, 3 \quad \text{respectively}$$

Here p_0 is the unknown pressure in front of the wave (in the solid body), Q_v is the heat of vaporization, and $(K_r)_w$ is the effective coefficient of reflection from the vaporization wave. It can only be determined from a theoretical or experimental study of the structure of the wave.

Since most of the energy is absorbed in the vapors and does not reach the vaporization wave, the value F_w^- is usually small compared to F_* and accordingly is smaller than F_*^+. Therefore $(K_r)_w$ is not a very important parameter, and for the concrete calculations carried out below values were used which were determined on the basis of the fact that in the absence of screening (when the effect of this parameter is not great) the pressure p_0 calculated at the surface of the solid body is close to the experimental pressure [7, 8]. In particular, for aluminum we took $(K_r)_w = 0.72$. In the derivation of (1.8) it is assumed that the radiant flux is $F = 0$ in front of the vaporization wave, and the enthalpy of the cold material in the solid state is equal to $(-Q_v)$. Using (1.8) and (1.3) we find that the following condition of balance of the radiant and hydrodynamic fluxes is satisfied at the critical point:

$$F_* + \dot{M}\left(h_* + \tfrac{1}{2}u_*{}^2\right) = -\dot{M}Q_v \tag{1.10}$$

Since a transition through the speed of sound occurs here,

$$u_* = c_* = \sqrt{K_* p_* / \rho_*} \tag{1.11}$$

Here c is the speed of sound and K is the differential adiabatic index:

$$K = (d \ln p/d \ln \rho)_{s=\text{const}} \tag{1.12}$$

2. The effective adiabatic index γ is determined from the equation

$$e = \frac{p}{\rho(\gamma - 1)}, \quad \text{or} \quad h = \frac{p\gamma}{\rho(\gamma - 1)} \tag{2.1}$$

Here e (or h) and p are taken as unknown functions of the temperature T and density ρ (or of the specific volume $v = \rho^{-1}$). It is useful to introduce two other dimensionless functions C and N through the equation

$$pv = NR'T, \quad h = CR'T; \quad R' = R/A_w \tag{2.2}$$

Here R is the universal gas constant and A_w is the atomic weight (in the un-ionized state).

It is natural that N, C, and γ are related to each other by the following equation:

$$C/N - 1 = (\gamma - 1)^{-1} \tag{2.3}$$

In an ideal, monoatomic un-ionized gas we have

$$N = 1, \quad C = \tfrac{5}{2}, \quad \gamma = \tfrac{5}{3}$$

From now on the equation of state is needed in the differential form. For this we differentiate (2.2) and obtain

$$\frac{dh}{h} - A_v \frac{dv}{v} - A_p \frac{dp}{p} = 0 \tag{2.4}$$

where

$$A_p = \left(1 + \frac{\partial \ln C}{\partial \ln T}\right)\left(1 + \frac{\partial \ln N}{\partial \ln T}\right)^{-1} \tag{2.5}$$

$$A_v = A_p\left(1 - \frac{\partial \ln N}{\partial \ln v}\right) + \frac{\partial \ln C}{\partial \ln v} \tag{2.6}$$

Here the partial derivative with respect to v is taken at $T = \text{const}$. For adiabatic movement the following relation occurs:

$$de + pdv = 0 \quad \text{or} \quad dh - vdp = 0 \tag{2.7}$$

From (2.1) and (2.7) we obtain a relation for K, γ, A_p, and A_V:

$$dh/h = (\gamma - 1)\gamma^{-1}dp/p \tag{2.8}$$

$$K = \gamma A_v[1 + (A_p - 1)\gamma]^{-1} \tag{2.9}$$

Thermodynamic functions usually have T and v as their arguments. However, it is convenient to conduct gas dynamic calculations (not only in the present problem) since there are tables of T and γ as functions of e and v (or h and v).

Let us now make some comments useful from a practical aspect. If the scale of the arguments of the functions of γ or $\varkappa$, given by the table at the points e_i and v_k, is nonuniform, the search for the desired entry in the tables is difficult. The desired entry is found by a successive sorting over of the arguments e_i and v_k and comparing them with some values of e_0 and v_0 for which the function of γ or $\varkappa$ must be determined. In addition, the total volume of work is significantly reduced (especially for a table with a large number of points) if the scale of the table is made uniform. Then, having taken whole fractions from the ratios $(e_0 - e_m)/\Delta e$ and $(v_0 - v_m)/\Delta v$, where e_m and v_m are the minimum values of the arguments and Δe and Δv are the steps (constant), we immediately obtain the numbers i and K of the desired entry. As for tables of thermodynamic and optical properties, a uniform scale is admissible (without an excess of points in some regions and their shortage in others) if e and v or their logarithms are taken as the arguments. In this case it is also easy to obtain an expression for the differential adiabatic index:

$$K = \gamma + \frac{\partial \ln \gamma}{\partial \ln e} - \frac{\partial \ln \gamma}{\partial \ln v}\frac{1}{(\gamma - 1)} \tag{2.10}$$

Here the partial derivative with respect to v is taken at $e = \text{const}$. Introducing

$$\chi = Q/h_*, \quad \eta = 1 + (\gamma_* - 1)K_*/2\gamma_* \tag{2.11}$$

Eq. (1.10) can be rewritten in the form

$$\dot{M}h_*(\chi + \eta) = -F_* \tag{2.12}$$

3. Dimensionless equations are obtained from the system of equations written out in Sec. 1 if dimensionless variables (designating them by the same letters and the upper index v) are introduced through the equations

$$p^v = p/p_*, \quad \rho^v = \rho/\rho_*, \quad r^v = r/r_*, \quad F^v = F/F_*,$$
$$u^v = u/u_*, \quad \varkappa^v = \varkappa/\varkappa_*, \quad (F^+)^v = F^+/F_*, \quad (F^-)^v = -F^-/F_* \tag{3.1}$$

Since we will use only the dimensionless variables here, we omit this index. As a result we obtain the following system:

$$dp + {}^1/_2 K_* \rho dg = 0, \quad g = u^2 \tag{3.2}$$

$$h + g(\eta - 1) + \chi = F(\eta + \chi) \tag{3.3}$$

$$\rho uS = 1, \quad S = r^{v-1} \tag{3.4}$$

The differential form of the equation of state (2.4) does not change its form after the transition to dimensionless variables. From (3.2)-(3.4) and (2.4) one can obtain

$$\frac{dS}{S}\left[\frac{dF}{dS}\frac{S}{F}\frac{F}{h}\frac{(\eta + \chi)}{A_v} - 1\right] = \frac{dg}{2g}\frac{(h - g\varphi)}{h} \tag{3.5}$$

The following notation was introduced here:

$$\varphi = \frac{\gamma}{K(\gamma - 1)}\frac{K_*(\gamma_* - 1)}{\gamma_*} \tag{3.6}$$

The equation (2.9) was used for the result (3.5).

At the sonic point we have

$$p = h = g = F = S = \varphi = 1 \tag{3.7}$$

For a continuous transition through the speed of sound it is necessary that the sonic point is singular. Therefore the following equation is involved:

$$dF/dS = A_v^*/(\eta + \chi) \tag{3.8}$$

It is easy to show that it relates the energy release with the work of the forces of expansion.

We note that Eq. (3.5) is obtained independently of the mechanism of energy transport (by monochromatic radiation, continuous spectrum radiation, or, for instance, electron thermal conduction). Therefore the condition of transport through the speed of sound (3.8) is also sufficiently general.

If the energy transport takes place only through monochromatic radiation, the dimensionless transport equations (1.4) and (1.5) take the form

$$\frac{dF^-}{dS} = \frac{\varkappa F^-}{\lambda (v - 1) g^{1/2} S^{\omega+1}} \tag{3.9}$$

$$\frac{dF^+}{dS} = - \frac{\varkappa F^+}{\lambda (v - 1) g^{1/2} S^{\omega+1}} \tag{3.10}$$

where $\lambda = l_*/r_*$, $\omega = (v - 2)/(v - 1)$ (l is the radiation range, $l = 1/(\varkappa\rho)$, $F^+ > 0$, $F^- > 0$, $F = (F^+ - F^-) > 0$). From this we obtain the following condition at the critical point:

$$\lambda (v - 1) = - (\chi + \eta)(F^+(1) + F^-(1))/A_v^* \tag{3.11}$$

Note that (3.11) involves not the total flux $F(1) = F^+(1) - F^-(1)$ but the sum $F^+(1) + F^-(1)$, the analog of the radiant energy density, since the condition (3.8) and (3.11) is a condition not on the flux but on the energy release at the critical point. This remark is important for the generalization of the solution considered here to the case of a continuous spectrum.

4. The analysis at the singular point can be obtained from (3.5) and (3.8)-(3.11) as well as from the original system. Introducing (for $|g-1| \ll 1$), as in [1],

$$F - 1 = z (g - 1), \quad S - 1 = A (F - 1) = Az (g - 1) \tag{4.1}$$

we obtain

$$A = (\chi + \eta)/A_v^* \tag{4.2}$$

It follows from (3.3) and (3.4) that

$$\rho - 1 = -(1/2 + Az)(g - 1)$$
$$h - 1 = [AA_v^*z - (\eta - 1)] (g - 1) \tag{4.3}$$

Near the singular point the approximation (0.1) can be used, where

$$\alpha = \alpha_* = \left(\frac{\partial \ln \varkappa}{\partial \ln h} \right)_*, \quad \beta = \beta_* = \left(\frac{\partial \ln \varkappa}{\partial \ln \rho} \right)_* \tag{4.4}$$

If $\varkappa$ is given as a function of h and ρ, then α and β are determined directly by numerical differentiation of the tables.

Therefore near the critical point we obtain the following expansion for the dimensionless $\varkappa$:

$$(\varkappa - 1) = - \alpha (h - 1) + \beta (\rho - 1) \tag{4.5}$$

If $\varkappa$ is given as a function of T and ρ, then

$$\alpha = - \frac{\partial \ln \varkappa}{\partial \ln T} \left(1 + \frac{\partial \ln C}{\partial \ln T} \right)^{-1}, \quad \beta = \frac{\partial \ln \varkappa}{\partial \ln \rho} + \alpha \frac{\partial \ln C}{\partial \ln \rho} \tag{4.6}$$

As a result, taking into account (3.11), we obtain from (3.5) a quadratic equation for determining the slope z of the integral curve:

$$a_z z^2 + b_z z + c_z = 0 \tag{4.7}$$

For simplicity we shall confine ourselves to the case when the change in φ can be neglected. Then the coefficients in (4.7) are determined in the following way:

$$a_z = \frac{2}{A_v{}^*}\left\{-1 + (\eta + \chi)\left[\alpha + \frac{(\beta + \omega)}{A_v{}^*} + 1\right]\right\} \tag{4.8}$$

$$b_z = [-2\alpha(\eta - 1) - (2\eta - 3) + \beta]/A_v{}^* + 1 \tag{4.9}$$

$$c_z = -\eta/(\eta + \chi) \tag{4.10}$$

It is easy to see that for $\gamma = K = \text{const}$ we obtain the following expressions for a_z, b_z, and c_z:

$$a_z = 2[-1 + (\alpha + \beta + \omega + 1)(\eta + \chi)] \tag{4.11}$$

$$b_z = -\alpha(\gamma - 1) + \beta - \gamma + 3 \tag{4.12}$$

$$c_z = -(\gamma + 1)/2(\eta + \chi) \tag{4.13}$$

We note that in [1], where the case of $\gamma = \text{const}$ was examined, an incorrect expression is given for a_z which, however, is practically not reflected in the results of the numerical calculations. The error was pointed out by B. M. Prosvirnina, for which the authors are grateful.

The problem can be solved in the following way. We shall set ourselves the temperature T_* and density ρ_* in the critical section. We find h_* and γ_* as well as K_*, α_*, and β_* from thermodynamic tables. We find the velocity of movement u_* from Eq. (1.11), and η and χ from Eq. (2.11). Let us set an arbitrary value for $F^+(1)$, for example, $F^+(1) = 0$. It is convenient to begin solving the series of variants with these parameters T_* and ρ_* since practically all the radiation is absorbed in the vapors and therefore in reality $F^+(1) \ll 1$. Subsequently decreasing T_* gradually, we will begin the calculation each time using the $F^+(1)$ from the preceding variant. We find the dimension r_* of the critical section from (3.11), and the total size of the flux F_* from (2.10). Using (4.7)-(4.10) we leave the singular point and begin a numerical calculation of the simultaneous differential equations, moving toward the surface of the body. A condition of phase equilibrium is observed immediately in front of the vaporization wave so that the temperature is related to the pressure through a single-valued dependence: $T_W = T_V(p)$. This dependence is usually approximated by the following analytical function:

$$\lg p = a - b/T \tag{4.14}$$

For aluminum according to a data analysis presented in [9], where the dependence $T_V(p)$ is obtained up to $p = 100$ bar, we have $a = 5.351$ and $b = 1.223$ if p is in bar and T in eV. After the dependence $T(p)$ obtained from the calculation intersects (4.14), we calculate the pressure P_0 in front of the vaporization wave from Eq. (1.7). Equations (1.6) and (1.8) are automatically satisfied since they were taken into consideration in the equations in the critical section. We obtain F_W^+ from Eq. (1.9). Since $F_*^+ = F_W^+(F_W^-/F_*^-)$, we immediately find F_*^+. Now we can start the calculation over again up to full convergence. A rather large number of iterations is usually required.

We then carry out a calculation of the supersonic zone, moving away from the body until F^+ (or F^-) no longer changes. This usually occurs at a distance of several critical section radii. We now determine q_0 from F_∞. From now on the movement can be considered as adiabatic and Eq. (1.3) is used to determine the flow velocity u_∞ (at infinity).

5. Tables of the thermodynamic and optical properties of vapors of the material which are needed to conduct such calculations are unfortunately nonexistent. Therefore these calculations were conducted for several materials.

The degree of ionization of the substance was determined from Saha's system of equations of ionization equilibrium [10] assuming the plasma to be electrically neutral and taking into account the law of conservation of nuclei per unit volume (equation of material balance) and the reduction in the ionization potential [10, 11]. This nonlinear algebraic system was solved by a method described in [12]. The internal energy e of the material was calculated as for an ideal gas with the Debye correction [10, 11], the electron excitation energy, and the energy loss in ionization taken into account. The energy levels and statistical weights needed for the calculation were taken from tables [12-14]. The change in the number of particles due to ionization and the Debye correction were considered in calculating the pressure. The precision in the calculation of the thermodynamic functions is evidently close to the precision in the corresponding calculations for air [15].
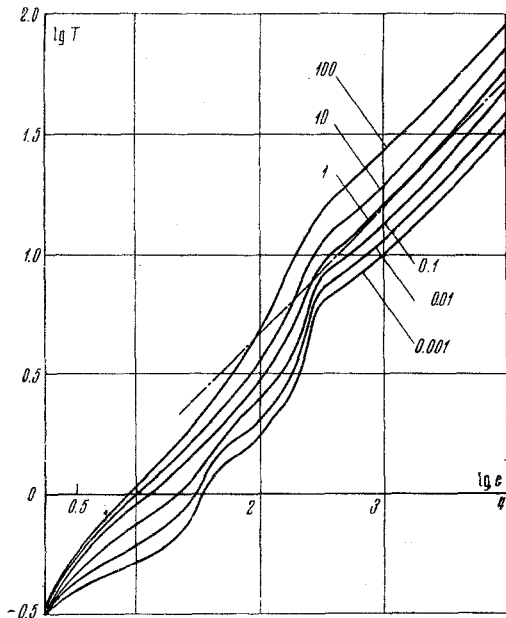
Fig. 1

In calculating the coefficients of absorption allowance was made for the free−free transitions of electrons in the field of ions and for absorption owing to bound−free transitions from highly excited states on the assumption that these states are hydrogen-like (in substituting summation over the levels taking part in the absorption for integration [16]). The resulting expression has the following form:

$$l = 0.571 \cdot 10^{-2}\, \varepsilon^{+3} T^{1/2} / \{[1 - \exp\,(-\varepsilon/T)]\psi\} \qquad (5.1)$$

Here l is in cm, the energy ε of the quanta and the temperature T are in eV, δ is the relative density, α_e is the relative electron concentration, and α_i is the relative concentration of ions having charge number i($i = 0$ for a neutral atom), and

$$\psi = \alpha_e \left(\sum_i \alpha_i i^2\right) \exp\,(\varepsilon/T)$$

$$\delta = \rho v_L = N_n / N_L, \quad \alpha_i = N_i / N_n, \quad \sum_{i=0}^{z} \alpha_i = 1 \qquad (5.2)$$

In the expression presented N_i is the number of particles per unit volume, N_n is the number of nuclei (or the "initial" number of particles "before ionization"), and N_L is the Loschmidt number. Hence the specific volume v_L (for $\delta = 1$) is equal to $2.2415 \cdot 10^4 / A_W$ cm^3/g. For aluminum $\rho_L = 1/v_L = 1.20 \cdot 10^{-3}$ g/cm^3.

For low temperatures T and degrees of ionization α_e the concentration of neutral atoms and the free−free transitions of electrons in the field of such atoms become important. They were accounted for in accordance with the results of [17]. The elastic interaction cross section needed for the calculations is unfortunately not known for metal vapors. Therefore it was rather roughly estimated from the "size of the atom" according to the hydrogen-like approximation and the first ionization potential I_1. For aluminum it was taken as equal to 20.4 πa_0^2, where $\pi a_0^2 = \sigma_B$ is the Bohr cross section.

In this temperature region the error introduced in this way is still evidently less than the inaccuracies connected with other assumptions, primarily with the assumptions of an ideal plasma, full equilibrium both of the phase transition and the degree of ionization, the absence of electron diffusion and electron thermal conduction, and also ignoring absorption in broadened lines.

The results of the calculations for approximately 80 temperatures (up to T = 40 eV) and 12 densities from $\delta = 100$ to $\delta = 3 \cdot 10^{-4}$ were then reinterpolated on a uniform scale for ln (e) (also at 80 points) and ln (v). We shall present the results of the calculations for aluminum.

The dependence of the temperature T (eV) on e (kJ/g) and δ (a grid for δ twice as fine was actually used in the calculations) is presented in Fig. 1, the dependence of the effective adiabatic index γ on e and δ is presented in Fig. 2, and the dependence of the mass absorption coefficient $\varkappa$ (cm^2/g) on e and δ for quanta with energy $\varepsilon = \varepsilon_0 = 1.16$ eV is presented in Fig. 3. (The values of $\varkappa$ on the phase equilibrium curve are shown by dashes.) The absorption coefficient for quanta of other energies $\varepsilon \neq \varepsilon_0$ can be estimated from the dependence shown in Fig. 3 (those for which an overall calculation of the absorption with highly excited states is permissible, i.e., with energies $\varepsilon = 1$-3 T):

$$\varkappa = \varkappa_0 \,(\varepsilon_0/\varepsilon)^3 \,[\exp\,(\varepsilon/T) - 1]/[\exp\,(\varepsilon_0/T) - 1] \qquad (5.3)$$

It is easy to see that $\varkappa \sim \varepsilon^{-2}$ for $\varepsilon \ll T$.

The dependences are analogous for other elements, although for elements with lower atomic numbers Z, such as Be and C, they have a less smooth nature, while conversely, the dependence is smoother for materials having higher Z (for example, W and Pb). Accordingly, the approximations (0.1) and $\gamma =$ const are applicable over a wider range for the latter.

6. A rough approximation of the tabular dependence presented in Fig. 3 in the region of multiple ionization leads to the following expression (if ε is in eV):

$$\varkappa = K\varepsilon^{-2}\delta^{0.6}, \quad K = 7.90 \cdot 10^6 \,\text{cm}^2/\text{g}, \quad K = K\rho_L^{-0.6} \qquad (6.1)$$
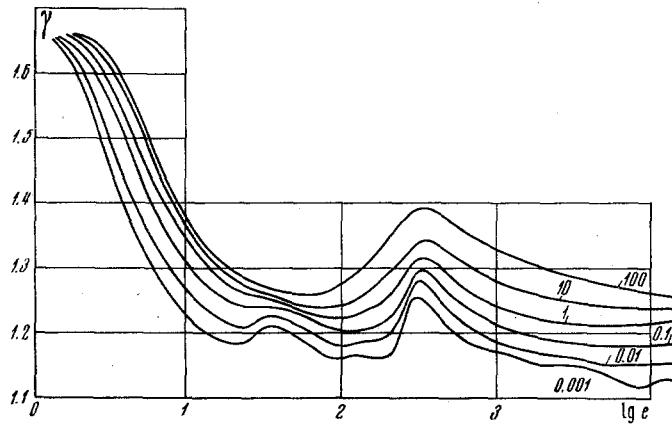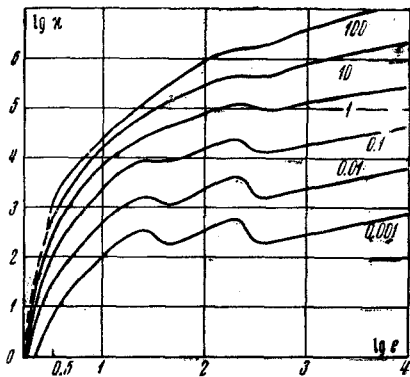
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Fig. 2

Fig. 3

In accordance with the results of the calculations presented in Fig. 2 one can take $\gamma = 1.20$ as a typical value. Using (6.1) it is easy to obtain approximate functions for the parameters both in the stage of the stationary state and in the preceding stage of the nonstationary plane wave of heating. The isomorphic solution [5, 6] can be used for this purpose. At the request of the authors V. M Krol' conducted calculations of a system of simultaneous differential equations describing the distribution of parameters in the isomorphic system and obtained

$$e_m = 25 q_0^{0.842}\, t^{0.526}\, \varepsilon_0^{-1.05}$$
$$p_0 = 8.6 q_0^{0.579}\, t^{-0.263} \varepsilon_0^{0.526}$$
$$u_m = 9.5 q_0^{0.421} t^{0.263}\, \varepsilon_0^{-0.526}$$
$$\rho_m = 0.50 \cdot 10^{-4} q_0^{-0.263} t^{-0.789}\, \varepsilon_0^{1.759}$$
$$m_w = 0.55 \cdot 10^{-4} q_0^{0.158}\, t^{0.474} \varepsilon_0^{1.05}$$

(6.2)

Here e_m is the maximum internal energy, and ρ_m is the density at the point where $e = e_m$. The quantity u_m is the maximum velocity of the vapors, m_w is the amount of vaporized mass, and p_0 is the pressure at the vaporizing surface. Having integrated the dependence $p_0(t)$ we obtain the following ratio of the impulse J created to the supplied energy E:

$$J/E = 12\ E^{-0.421} t^{0.158} \varepsilon_0^{0.526} = 12\ E^{-0.263} q_0^{-0.158} \varepsilon_0^{0.526}$$

(6.3)

Here and afterward q_0 is in MW/cm², u in km/sec, t in μsec, e in kJ/g, p in bar, ρ in g/cm³, m in g/cm², E in J, and J is in dyn·sec.

Approximating the dependence of the internal energy on the temperature T (eV) and the relative density δ presented in Fig. 1 in the following way:

$$e = 5.37\ T^{1.90} \delta^{-0.154}$$

(6.4)

(the dash-dot line in Fig 1 for $\delta = 1$) we obtain an expression for the maximum temperature (in eV):

$$T = 1.5\ q^{0.442} t^{0.213} \varepsilon_0^{0.410}$$

(6.5)

Let us now examine the stationary state, using the conservation laws at the critical point and the condition (3.11), which in the absence of reflection, with $\gamma = \text{const}$, and without taking into account heat expended in vaporization, i.e., for $Q_V = 0$, converts into $\lambda = l_* / r_* = (\gamma + 1)/4$; we obtain

$$\rho_* = 1.453\, (Kr_*)^{-5/8}\, \varepsilon_0^{5/4}$$
$$e_* = 1.04 q_*^{2/3} (Kr_*)^{5/12} \varepsilon_0^{-5/6}$$
$$p_* = 0.303 q_*^{2/3} (Kr_*)^{-5/24} \varepsilon_0^{5/12}$$
$$u_* = 0.50 q_*^{1/3} (Kr_*)^{5/24} \varepsilon_0^{-5/12}$$

(6.6)

A calculation of the simultaneous differential equations describing the distribution of the parameters in the stationary state, assuming the correctness of (0.1), led to the following values of the dimensionless parameters:

$$F_\infty{}^v = 1.48, \quad e_m{}^v = 1.15, \quad u_\infty{}^v = 3.26, \quad p_w{}^v = 2.29, \quad r_0{}^v = 0.815$$

Inserting these values and considering that $q_* = 0.448\, q_0$ and $r_* = 1.227\, r_0$ we find

$$e_m = 26.5 q_0^{0.667} r_0^{0.416} \varepsilon_0^{-0.834}$$
$$p_0 = 6.6 q_0^{0.667} r_0^{-0.209} \varepsilon_0^{0.416}$$
$$J/E = 6.6 q_0^{-0.333} r_0^{-0.208} \varepsilon_0^{0.416} \tag{6.7}$$
$$\rho_* = 0.63 \cdot 10^{-4} r_0^{-0.625} \varepsilon_0^{1.25}$$
$$u_\infty = 7.9 q_0^{0.333} r_0^{0.208} \varepsilon_0^{-0.416}$$
$$T_m = 1.8 q_0^{0.351} r_0^{0.168} \varepsilon_0^{-0.338}$$

Here the sphere size r_0 is in cm.

For comparison we shall present analogous dependences for a fully ionized gas, to be specific, beryllium ($\alpha_e = Z = 4$, which is approximately equal to the average value of α_e for aluminum in the temperature range considered). We borrow the values of the characteristic dimensionless parameters from [1, 6]. In the law (0.1) if $\varkappa$ is in cm²/g, ρ in g/cm³, and e in kJ/g the constant $K = 0.50 \cdot 10^{14}$. The exponents are $\alpha = \frac{3}{2}$, $\beta = 1$. For a plane nonstationary wave of heating we obtain

$$p_0 = p_w = 5.6 q_0^{3/4} t^{-1/8} \varepsilon_0^{1/4}$$
$$\rho_m = 0.26 \cdot 10^{-5} q_0^{1/4} t^{-3/8} \varepsilon_0^{1/4}$$
$$e_m = 105 q_0^{1/2} t^{1/4} \varepsilon_0^{-1/2} \tag{6.8}$$
$$m_w = 0.5 \cdot 10^{-5} q_0^{1/2} t^{3/4} \varepsilon_0^{1/2}$$
$$u_0 = 54 q_0^{1/4} t^{1/8} \varepsilon_0^{-1/4}$$
$$J/E = 6.3 q_0^{-1/4} t^{-1/8} \varepsilon_0^{1/4}$$
$$T = 1.5 q_0^{1/2} t^{1/4} \varepsilon_0^{-1/2}$$

Note that the energy expended on the complete ionization of beryllium is equal to $\sim 4 \cdot 10^3$ kJ/g while this state of the vapors begins at a temperature of ~ 50 eV. In the quasistationary state of radially symmetrical movement we obtain the following variation functions of the parameters:

$$\rho_* = 0.40 \cdot 10^{-5} q_0^{1/3} r_0^{-1/3} \varepsilon_0^{2/3}$$
$$e_m = 61 q_0^{4/9} r_0^{2/9} \varepsilon_0^{-4/9}$$
$$p_0 = p_w = 4.8 q_0^{7/9} r_0^{-1/9} \varepsilon_0^{2/9} \tag{6.9}$$
$$u_0 = 30 q_0^{2/9} r_0^{1/9} \varepsilon^{-2/9}$$
$$J/E = 4.8 q_0^{-2/9} r_0^{-1/9} \varepsilon_0^{2/9}$$
$$T_m = 0.88 q_0^{4/9} r_0^{2/9} \varepsilon_0^{-4/9}$$

As follows from a comparison of the functions (6.3) and (6.8) and of (6.7) and (6.9), in the region of multiple ionization we have a sharper dependence of the parameters on the flux density q_0 of the incident radiation and the sphere size r_0 (and in the plane case on the time t) as well as on the energy ε of the quanta than in the case of a fully ionized gas [1, 5, 6].

7. The exact numerical calculations of the problem involving the stationary state in which the functions $\varkappa(e, \rho)$, $\gamma(e, \rho)$ and $T(e, \rho)$ were used allow the results presented above to be made more precise and explain the degree of deviation from the indicated functions such as (6.7) and (6.9) because of a difference from the law (0.1) or in the case of aluminum from (6.1).

A series of calculations was carried out for a number of materials at several values of the relative density δ_* in the critical section and for different temperatures T_* (from 1.75 to 45 eV). The tables used for aluminum are presented in graph form in Figs. 1-3.

We note that according to (6.1) and the condition (3.11), for a single value of δ_* the dimension r_* (and r_0) should depend weakly on the temperature T_*.

The results of the calculation for Al vapors at $\delta_* = 0.1$ and three temperatures T_*, namely 2, 3, and 4 eV, are presented in Figs. 4-6. (Note that the index is omitted for the dimensionless variables.) Let us show to what dimensional parameters these variants correspond. The sphere size was 0.50, 0.40, and 0.30 cm, respectively, the dimension of the critical section was 1.0, 0.66, and 0.44 cm, the speed of sound in the critical section equalled 4.4, 6.2, and 8.1 km/sec, the time for establishing the stationary state was 61, 28, and 17 nsec, and the velocity of flow of the vapors was 15, 20, and 25 km/sec.
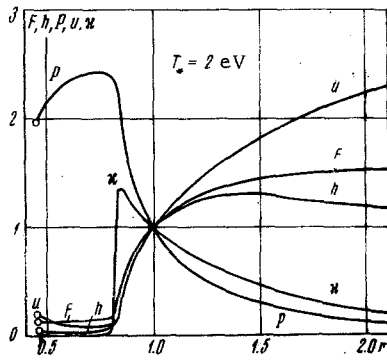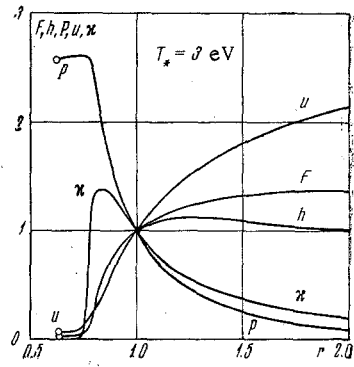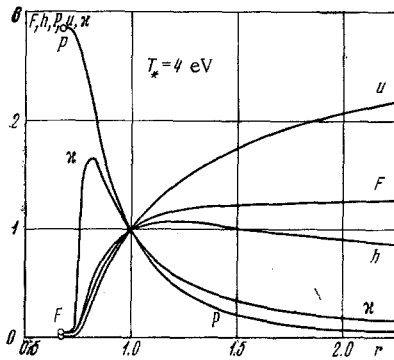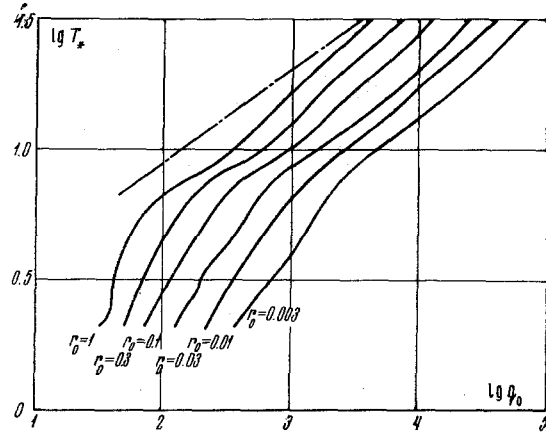
Fig. 4

Fig. 5

Fig. 6

Fig. 7

The pressure p_0 at the surface was 57, 100, and 170 bar; the pressure p_* at the critical point was 19, 36, and 59 bar; the flux density q_0 of the radiation at the sphere without shielding was 43, 56, and 90 MW/cm^2; and the radiation flux density q_* at the critical point was 6, 16, and 31 MW/cm^2.

The presence of a rather extended zone of relatively cold vapors at the surface of the solid body attracts attention. The gas in this zone is weakly ionized and absorbs almost none of the radiation passing through the hot ionized layer which screens the vaporizing surface.

It should be noted that the heating of the vapors takes place very abruptly and one can speak of the presence of a front of the wave of heating. This is related to the rather rapid growth in the coefficient of absorption with the temperature (see Fig. 3).

As follows from Figs. 4-6, the thickness of the cold zone decreases with a growth in T_* and δ_*. The latter is evidently connected with the increase in pressure p_W at the surface and, in accordance with (4.14), in the temperature of the phase transition and the coefficient of absorption $\varkappa_W$ near the evaporating surface.

We also note the nonmonotonic nature of the change in pressure and velocity: in drawing away from the surface the gas is slowed and the pressure increases [as in an ordinary (without preheating) subsonic nozzle having diverging walls], and only then does it begin to accelerate again in the subsonic zone, but there the absorption of radiation and the release of energy by the vapors become important, while the pressure drops (as in a pipe of constant cross section for a heated gas). In the supersonic zone the pattern does not differ qualitatively from that which is obtained by calculations [1] assuming the correctness of (0.1).

If the dependence of the coefficients (4.11) and (4.13) in the quadratic equation (4.7) on α and β is analyzed, noting that β always lies within the range of 0-1, it is found that no solution is possible for this equation for any negative value of α, and therefore it is not always possible to find an integral curve passing through the singular point in the desired fashion [1]. Since in accordance with Fig. 3 the coefficient α in Eq. (0.1), which can be used as an approximate equation near the singular point, also gradually decreases and becomes negative with a decrease in T_*, it can be expected that a solution cannot be successfully constructed for low temperatures T_*. This is also confirmed by the fact that (as seen in Figs. 4-6) as T_* de-
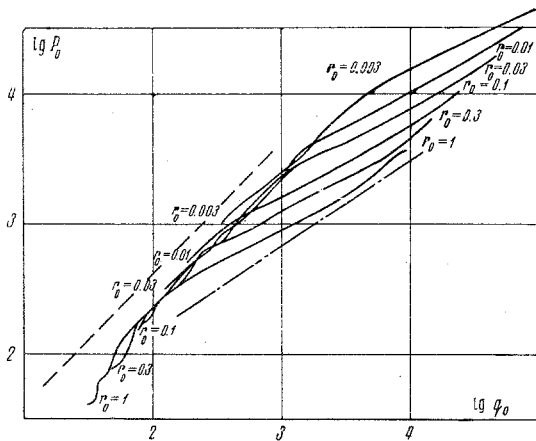
Fig. 8

creases, the sonic point approaches the maximum point of the dimensionless coefficient of absorption $\varkappa^V = \varkappa/\varkappa_*$. For $T_* \lesssim 1.75$ eV it was actually impossible to construct a solution. However, the reason was not the nature of the coefficients in (4.7) indicated above but the fact that still earlier a second singular (sonic) point appears at the vaporizing surface. This is related to the increase in the velocity of the vapors in the cold zone as the surface is approached. Analyzing the dependence of the number M_w at the vaporizing surface for a fixed T_* we discover that for some value of T_* the number $M = u_w / c_w$ reaches unity.

The states which occur with a further decrease in T_* were not studied. It is possible that discontinuous flow develops here with a shock wave closing the first supersonic zone, and it is possible that a solution of the problem for the stationary state does not exist at all for these parameters.
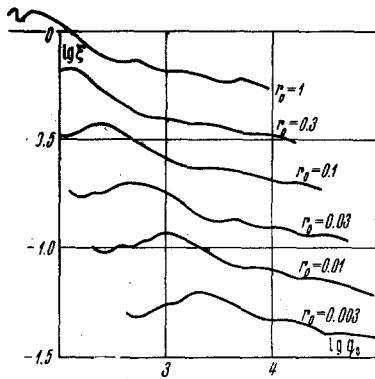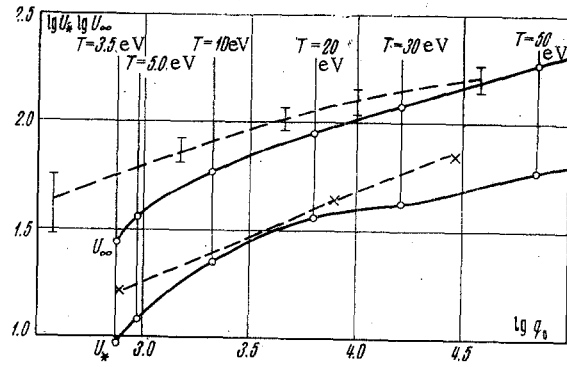
It is necessary to keep the following in view: since under actual conditions of the action of the radiation a nonstationary phase of "flash" absorption occurs, the solution of the quasistationary state problem can be used only for estimating the time-averaged parameters (as shown by calculations for the nonstationary problem conducted by V. I. Bergel'son in a variation of the method of [2, 3], and after the time $t_* = r_*/c_*$ of "establishing the stationary state" some pulsations occur in the vaporization carrying the disturbance also to the "hot" region of the vapors).

The results of the calculations for a given entry of T_* and δ_* values were reinterpolated to a given entry of q_0 and r_0 values.

The dependence of the temperature T on q_0 and r_0 is presented in Fig. 7. It is easy to see that this dependence can be described by the exponential function (6.7) only for sufficiently high T_* and q_0 (dash-dot line in Fig 7 and following figures for $r_0 = 1$ cm). For large r_0 and low values of T_* (and q_0) the appearance of a region of two-valued and three-valued solutions is noted. This is related to the two-valued and three-valued nature of the dependence of $\varkappa$ on T (or e) for a fixed δ (see Fig. 3).

The dependence of the pressure p_0 at the surface of the solid body on q_0 and r_0 is presented in Fig. 8, and in this case the results of the numerical solutions are in good agreement with (6.7) also only for large q_0. As seen from Fig. 8, because of the effect of screening of the vaporizing surface by vapors the dependence of p_0 on q_0 stops being linear and becomes weaker. The ratio p_0/q_0 decreases with an increase in q_0 in comparison with the case of the absence of screening (dashed line).

The dependences of the parameters found also permit an estimate of the intensity of reradiation by the hot vapors. Let us turn our attention first of all to the fact that according to (5.3) the coefficient of absorption $\varkappa$ of quanta of energy ε exceeding energy ε_0 of the incident radiation quanta is less than $\varkappa(\varepsilon_0) = \varkappa_0$. Thus, the emitted radiation is volumetric. Equation (5.3) is valid, generally speaking, only for quanta with energies of $\varepsilon \lesssim 3T$ [10, 16], i.e., for T = 7 eV, up to $\varepsilon = 20$ eV. In the region of harder quanta it is impossible to employ a summation of all the levels; rather they must be taken into account individually, since the levels absorbing such radiation are clearly not hydrogen-like. Nevertheless, to obtain a very preliminary and round approximate estimate we can extend (5.3) down to values of ε_i equal to the typical ionization potential I.

The intensity of the volumetric radiation of a unit mass is equal to

$$ f = 4 \int_0^{\varepsilon_i} B_\varepsilon \varkappa_\varepsilon \, d\varepsilon, \quad B_\varepsilon = \frac{15}{\pi^4} \frac{\sigma \varepsilon^3}{\exp(\varepsilon/T)-1}, \quad \int_0^\infty B_\varepsilon \, d\varepsilon = \sigma T^4 \tag{7.1} $$

Here B_ε is the Planck function, σ is the Stefan–Boltzmann constant ($\sigma = 0.1029$ MW/cm$^2 \cdot$ eV^4). Using (5.3) and the assumption concerning its applicability made above we obtain

$$ f = 60\pi^{-4}\sigma\varepsilon_0{}^2 T^2 \varepsilon_1 / T \tag{7.2} $$

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Fig. 9

Fig. 10

At the same time, the intensity of energy release due to laser radiation absorption is approximately equal to $\varkappa_0 q_0$. Substituting these values we obtain an approximate criterion for the role of reradiation:

$$\xi = 1.20 \cdot 10^{-2} \; T^2 \varepsilon_0{}^2 / q_0 \tag{7.3}$$

Here $\varepsilon_1 = 7T$ (according to [10] and the calculation the typical values vary within the limits of 5–10T).

Values computed according to (7.3) are presented in Fig. 9. As seen, for large sizes r_0 (starting with about 0.3 cm) the role of reradiation can be rather significant. The effect of reradiation can be two-fold: on the one hand part of the energy is generally lost in leaving the dense zone of vapors; on the other hand part is directed toward the surface of the solid body and is absorbed in the cold vapors, either reaching the vaporization wave or heating the cold layer near the vaporizing surface. It is proposed that these effects be analyzed in more detail in a separate article. Here we only indicate that a very effective means for such an analysis is the use of the problem of the stationary state examined here which is easily generalized to the case of energy transfer by continuous spectrum radiation. Let us also note that a very approximate estimate of the role of reradiation can be derived from [18] or from (6.3), (6.5), or (6.7), replacing the energy ε_0 of the quanta of incident radiation in these equations with the average energy $\bar{\varepsilon}$ of the quanta emitted by the vapors, which is determined at the maximum attained temperature T_q which is related to the radiation flux density q by Eq. (7.3) at $\xi = 1$. In this case, however, one should have in view the possibility of secondary reradiation from the zone heated by radiation of the continuous spectrum (tongue), since this radiation is closer to black body radiation with a temperature in the tongue of T_1 than to the radiation of the hot layer.

8. Let us compare our calculations with the experimental results of N. G. Basov, O. N. Krokhin, G. V. Sklizkov, et al. [19, 20].

The calculated dependences of the maximum flow velocity u_∞ for $r_0 = 0.1$ cm are shown by solid lines in Fig. 10. The experimental points (for a duration of the effect of $t_q = 15$–30 nsec), the results of measurement of the maximum flow rate of carbon vapors, are joined by dashed lines. The velocity measurements were made far enough from the surface so that the velocity was already almost constant and usually after the end of the laser's effect. However, the comparison with the calculations is not too improper if one considers that when the stationary state was established the velocity at the margin of the vapors is practically unchanged after the source is turned off and, as in the nonstationary problem, corresponds to the maximum concentration of the energy released.

The velocity u_* (lower solid curve in Fig. 10) is compared with the velocity of the boundary of the opaque region (lower dashed line). Note that according to the calculations conducted it is just near the critical point that the radiation begins to be strongly absorbed. The supersonic part of the jet is almost transparent.

As seen, there is satisfactory agreement of the calculated and experimental curves.

The temperatures attained are also plotted in Fig. 10. We note that at $T_* = 40$–50 eV carbon vapors are ionized down to the K shell.

As follows from Fig. 10, in the experiment the screening of the vaporizing surface and the increase in the vapor velocity connected with it were observed in a region of lower q_0 than follows from the calculations. This is evidently connected with the effect of the nonequilibrium nature of the ionization. Esti-

mates made by one of the authors together with S. P. Popov show that the electron temperature is somewhat higher than the ion temperature, and accordingly the degree of ionization α_e is also higher (especially in the region of low α_e and T). This leads to a somewhat higher absorption coefficient $\varkappa_w$ for the vaporizing surface and hence to an earlier development of screening and a thinner cold layer than follows from the calculations which were conducted on the assumption of thermodynamic equilibrium. Note that the nonequilibrium ionization, reducing the time for heating the vapors, exerts a stabilizing effect on the process of "burning" of the material (reradiation plays the same role), decreasing the duration of the pulsations detected in [2, 3]. The results of calculations which take this circumstance into account are published separately.

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